

Magnetic Screening in Thermal Yang-Mills Theories

T. S. BIRÓ

Institut für Theoretische Physik, Justus-Liebig Universität,
Heinrich-Buff-Ring 16, D-6300 Gießen, Germany

and

B. MÜLLER

Department of Physics, Duke University, Durham, NC 27708-0305

Abstract

We develop a semiclassical method to calculate the density of magnetic monopoles in non-abelian gauge theories at finite temperature in the dilute gas approximation. This quantity is related to the inverse magnetic screening length for which we obtain $\mu_M = 0.255g^2T$ in SU(2).

1. Introduction

The screening of static magnetic gauge fields in Yang-Mills theories at high temperature has long proved to be intractable by analytic techniques. There exist well known arguments¹ that the inverse screening length μ_M must be of order g^2T , and this has been partially supported by numerical calculations in the framework of lattice gauge theory² which have yielded the result $\mu_M = (0.27 \pm 0.03)g^2T$ for SU(2). The recently developed

resummation techniques³ for the finite-temperature gauge theories cure many infrared divergences arising in the perturbative expansion, but not those associated with long-range static magnetic gauge fields. Although the Schwinger-Dyson equation that determines μ_M has been identified some time ago⁴, its solution remains unknown. A recent attempt⁵ to calculate magnetic screening to leading order in dual QCD has led to the result $2\pi T/g$ for the dual gluon mass in the high-temperature limit, which is not directly related to the static magnetic screening mass.

Here we propose a new approach to the calculation of μ_M , which is not based on a perturbative expansion in the gauge coupling constant g . Our starting point is the observation* that the combination $g^2 T$ defines an inverse length without any factor involving powers of Planck's constant \hbar . One may therefore speculate that μ_M can be calculated from *classical* Yang-Mills statistical mechanics, with quantum effects providing corrections of higher order in g . This conjecture was recently found to be true for the thermal gauge boson (plasmon) damping rate, which also is of order $g^2 T$ and can be calculated either by resummation techniques⁶ or by a stability analysis of time-dependent classical gauge field configurations⁷. This coincidence is nontrivial, because in the perturbative calculation the gluon damping rate is seen to superficially depend on the electric screening mass, which is of order $gT/\sqrt{\hbar}$. However, this dependence exactly cancels in the leading order result.

Similarly, it has been emphasized by Landsman⁸ that the effective high-temperature effective action for the static sector of gauge theories contains quantum corrections describing the Debye screening of static electric fields. These effectively convert the dimensionally reduced Yang-Mills action into a Yang-Mills–Higgs action that remains unbroken at tree level. Magnetic monopoles can induce a symmetry breaking term into the effective Higgs action, as pointed out by Oleszczuk and Polonyi⁹. Our treatment assumes that these

* Note that in our convention g is defined by the *classical* Yang-Mills equations, and is related to the dimensionless coupling constant by $\alpha_s = g^2 \hbar / 4\pi$.

effects of the full quantum theory do not contribute to the magnetic screening mass at leading order in g . Whether that is indeed so will require further investigation.

This paper is organized as follows: we connect the magnetic screening mass in high-temperature Yang-Mills theories with the magnetic monopole charge density in the framework of the linear response theory. We then calculate the density of magnetic monopole charges from the canonical partition function in Chapter 3. A saddle point evaluation in the space of general radial symmetric monopole field configurations becomes possible by inserting an appropriate unit factor, which sets the scale and stabilizes classical soliton solutions of a given size. The integration over scale sizes is done at the end. Besides these stabilized classical field configurations the contributions of fluctuation modes with zero frequency, i.e. the collective modes of translation and rotation, must be taken into account because they determine the entropy of a given field configuration. Finally, in Chapter 4, we conclude by exploring the characteristic size and energy of dominant monopole configurations under conditions that may be reached in heavy-ion collisions at RHIC energies.

2. Magnetic Screening

In order to obtain the magnetic screening length one usually investigates the linear response of the medium to an infinitesimal external magnetic field. The magnetic field, which couples to the monopole charge, is curl-free and can be expressed by a scalar magnetic potential ϕ^a ,

$$B_i^a = -\partial_i \phi^a. \quad (1)$$

The application of such an external magnetic field modifies the monopole charge density in the plasma, so the partition function becomes

$$Z(\phi) = \text{Tr} e^{-\beta(\hat{H} - \phi^a \hat{Q}^a)}, \quad (2)$$

where \hat{Q}^a is the magnetic charge operator and the trace “Tr” runs over all possible states of the Yang-Mills field. Here we discuss how to evaluate this partition function in the case

of SU(2) only, but the result can easily be generalized to SU(N). Let us denote a given color charge state by $|q, q_3\rangle$, where q , the eigenvalue of the Casimir operator, is a multiplet index. A magnetic monopole state corresponds to the eigenvalue $q = 1$.

Since the Yang-Mills action is color symmetric the eigenstates of the Hamiltonian are q_3 independent, so one can always “rotate” an energy and color eigenstate $|\omega, q_3\rangle$ so that the action of $\beta\phi^a\hat{Q}^a$ can be represented by that of the generator of the abelian subgroup $\Theta\hat{I}_3 \equiv g^{-1}\beta|\phi|\hat{I}_3$, where g^{-1} is the elementary magnetic charge, \hat{I}_3 has the eigenvalues $q_3 = -1, 0$ and $+1$ and finally

$$|\phi|^2 = \phi^a \phi^a. \quad (3)$$

Using this particular representation of gluon states \hat{I}_3 and \hat{H} commute and in the dilute gas approximation the quantum numbers ω and q_3 are additive, so the partition function (2) factorizes for different ω and q_3 values.¹⁰ Taking into account the bosonic nature of excitations of the Yang-Mills field, the magnetic monopole partition function in the dilute gas limit becomes:

$$Z = \prod_{\omega} \prod_{q_3} \sum_{n=0}^{\infty} e^{-n(\beta\omega - q_3\Theta)}, \quad (4)$$

where q_3 runs over the possibilities $-1, 0$ and $+1$.

The summation over the indefinite occupation number n yields a Bose-Einstein factor for each ω and q_3 . From this partition function we obtain the magnetic color charge density using the standard formula

$$\rho^a = \frac{1}{\beta V} \frac{\partial}{\partial \phi^a} \ln Z. \quad (5)$$

In the weak external field limit ($\Theta \ll 1$) this leads to (see Appendix A for details):

$$\rho^a = -\frac{N}{Vg^2T} Z_M \phi^a \quad (6)$$

for the group SU(N) with the effective one-monopole partition function

$$Z_M = \sum_{\omega} \frac{1}{e^{\beta\omega} - 1}. \quad (7)$$

Because of the large average mass of a monopole we can make use of the Boltzmann approximation, obtaining

$$Z_M \approx \sum_{\omega} e^{-\beta\omega} = \text{Tr} \left(e^{-\beta\hat{H}} \right). \quad (8)$$

Since the magnetic monopole charge density is the source of the divergence of the magnetic field

$$\partial_i B_i^a = \rho^a, \quad (9)$$

we arrive at the following equation describing the linear polarizability of a magnetic monopole gas

$$\partial_i \partial_i \phi^a - \frac{N Z_M}{g^2 T V} \phi^a = 0. \quad (10)$$

Inspecting this equation one easily realizes that an effective magnetic screening mass is obtained,

$$\mu_M = \left(\frac{N Z_M}{(g^2 T)^3 V} \right)^{\frac{1}{2}} g^2 T, \quad (11)$$

causing an exponential damping of the external magnetic potential

$$\phi^a(r) \sim e^{-\mu_M r}. \quad (12)$$

It is reasonable to expect that the ratio Z_M/V scales with T^3 at high temperatures because T is the only available scale. Recall furthermore that the four-dimensional gauge theory at high temperatures undergoes dimensional reduction and becomes equivalent to a three-dimensional gauge theory with the effective temperature $g^2 T$. We therefore expect that the partition function scales as $Z_M \sim V(g^2 T)^3$, so the magnetic screening mass is proportional to $g^2 T$.

It is illuminating to repeat the same considerations for the electric sector of the gauge theory. In this case the external potential must be taken as that of a static color-electric field, and one considers the dilute gas of gauge field excitations with electric charge in the

adjoint representation. In lowest perturbative order these are just the free perturbative gauge bosons, but it is now inappropriate to use the Boltzmann approximation because there is no mass gap. As shown in Appendix A, this consideration yields precisely the electric screening mass $\sqrt{N/3}gT$ obtained in diagrammatic perturbation theory.

3. Partition Function of Magnetic Monopoles

3.1. SPHERICALLY SYMMETRIC GAUGE FIELDS

In order to calculate the nontrivial factor we need to obtain the partition function Z_M . Although simple scaling arguments^{1,11} show that there may be a characteristic monopole size R_0 contributing dominantly to the partition sum

$$Z_M \sim \int \frac{dR}{R^5} \exp(-R_0/R), \quad (13)$$

no classical monopole solution exists, which would offer a stable stationary point of the path integral defining the partition function.

While the t'Hooft-Polyakov solution¹² for spontaneously broken SU(2) gauge theories is stable and has finite energy, its counterpart in pure SU(2) gauge theory, the Wu-Yang monopole¹³, has infinite energy and is unstable against small perturbations^{14,15}. This lack of a basis for a semiclassical analysis of the statistical mechanics of monopole solutions in non-abelian gauge theories apparently vitiates an analytical approach. Here we want to sketch how the stability problem may be circumvented, opening the way to a semiclassical calculation of the magnetic monopole density at high temperature, and therefore of the magnetic screening length.

Let us begin by considering SU(2) gauge field configurations which carry one unit of magnetic charge. They must asymptotically look like a monopole ($j = 0$) mode of the operator

$$\hat{J} = \hat{S} + \hat{I} + \hat{L}, \quad (14)$$

where \hat{S} and \hat{I} denote the generators of spin and color spin in the adjoint representation and \hat{L} is the generator of orbital angular momentum for the gauge field. Since the gauge field belongs to the representation $S = I = 1$, there are three different possibilities to construct a $j = 0$ mode; namely $|L, T\rangle = |1, 1\rangle$, $|0, 0\rangle$, and $|2, 2\rangle$ combinations, where the grand-spin quantum number T is obtained from the eigenvalue of the Casimir operator $\hat{T}^2 = T(T + 1)$ with

$$\hat{T} = \hat{S} + \hat{I}. \quad (15)$$

The Wu-Yang monopole, known to be unstable, belongs to the $j = 0$ mode of type $|L, T\rangle = |1, 1\rangle$. A further unstable mode has been found to involve a combination of the two other states by Akiba, Kikuchi and Yamagida¹⁶.

Having this in mind we start our investigation with the most general ansatz¹⁷ for the monopole vector potential

$$A_{ia} \equiv \sum_{\alpha=\pm,0} P_{ia}^{(\alpha)} A^{(\alpha)} = \frac{1}{r} (P_{ia}^+(ue^{i\phi} - i) + P_{ia}^-(ue^{-i\phi} + i) + P_{ia}^0 w), \quad (16)$$

where $u(r)$, $\phi(r)$ and $w(r)$ are real functions of the radial variable only and we use the projectors

$$\begin{aligned} P_{ia}^{\pm} &= \frac{1}{2}(\delta_{ia} - n_i n_a \pm i\epsilon_{iaj} n_j), \\ P_{ia}^0 &= n_i n_a \end{aligned} \quad (17)$$

with the unit radial vector $n_i = x_i/r$. Configurations of the Wu-Yang monopole-type are proportional to $\frac{1}{2i}(P_{ia}^+ - P_{ia}^-) = \epsilon_{iaj} n_j$, i.e. they belong to the choice $w = 0$ and $u = 0$. The unstable mode found by Akiba et al. is proportional to a linear combination of $\frac{1}{2}(P_{ia}^+ + P_{ia}^-)$ and P_{ia}^0 , hence corresponds to $w \neq 0$ and $\sin \phi = 0$.

The different components of this ansatz, A^+ , A^- and A^0 , the respective coefficients of the projectors, are related to the $|L, T\rangle$ specification of the monopole mode ($j = 0$) by

simple linear combinations

$$|0, 0\rangle = \frac{2}{3}(A^+ + A^-) + \frac{1}{3}A^0, \quad (18)$$

$$|1, 1\rangle = \frac{i}{2}(A^+ - A^-) \quad (19)$$

and

$$|2, 2\rangle = \frac{1}{3}(A^+ + A^-) - \frac{1}{3}A^0. \quad (20)$$

The magnetic field described by our ansatz is

$$B_{ia} = \frac{1}{r^2} [P_{ia}^+ (iru' + u(r\phi' + w)) e^{-i\phi} + c.c. + P_{ia}^0 (1 - u^2)] \quad (21)$$

where the prime denotes radial differentiation and *c.c.* stands for complex conjugate.

The magnetic monopole charge seen from outside a sphere of radius r can be obtained from the magnetic analogue of Gauss' law

$$Q_a(r) = \frac{1}{4\pi} \int d^3r \partial_i B_{ia} = \frac{1}{4\pi} \oint_r n_i B_{ia} = n_a (1 - u^2(r)). \quad (22)$$

This result shows that a monopole field configuration requires asymptotically $u \rightarrow 0$ as $r \rightarrow \infty$, irrespective to the fields $w(r)$ and $\phi(r)$.

The energy of a static configuration in the high- T limit defines the effective action of the dimensionally reduced euclidean field theory

$$S_3[A] = \beta E = \frac{1}{g^2 T} \int d^3r \frac{1}{2} (E_{ia} E_{ia} + B_{ia} B_{ia}). \quad (23)$$

Introducing the scaled parameter $\tilde{\beta} = \frac{4\pi}{g^2 T}$, we find for the ansatz (16):

$$\begin{aligned} \beta E &= \tilde{\beta} \int_0^\infty dr \left[\left(\frac{du}{dr} \right)^2 + u^2 \left(\frac{d\phi}{dr} + \frac{1}{r} w \right)^2 + \frac{(1 - u^2)^2}{2r^2} \right] \\ &= \tilde{\beta} E[u] + \tilde{\beta} \int_0^\infty dr u^2 \left(\frac{d\phi}{dr} + \frac{w}{r} \right)^2. \end{aligned} \quad (24)$$

Here one observes that only the field $u(r)$ is really a dynamical degree of freedom while $w(r)$ is non-dynamical and $\phi(r)$ is a cyclic variable. Their physical interpretation becomes clear inspecting infinitesimal gauge transformations of the vector potential

$$\delta A_{ia} = D_{iab} \delta \Lambda_b, \quad (25)$$

where

$$D_{iab} = \delta_{ab} \partial_i - \epsilon_{acb} A_{ic} \quad (26)$$

is the gauge-covariant derivative. Any gauge transformation which conserves the monopole form of our ansatz (16) must have the form

$$\delta \Lambda_b = n_b \cdot \delta \Lambda(r). \quad (27)$$

This implies that restricting ourselves to static magnetic monopole gauge field configurations there still remains a residual gauge degree of freedom. The variation of the ansatz fields under such an infinitesimal gauge transformation is given by

$$\begin{aligned} \delta u &= 0, \\ \delta \phi &= -\delta \Lambda, \\ \delta w &= r \frac{d}{dr} \delta \Lambda. \end{aligned} \quad (28)$$

The field $u(r)$ and hence the monopole charge is unchanged, but the field ϕ is rotated by a gauge transformation mixing fluctuations of the pure Wu-Yang ansatz with the other unstable mode. The meaning of the field $w(r)$ is less obvious, we note only that it vanishes in the Schwinger gauge $x_i A_{ia} = 0$.

3.2. THE FUNCTIONAL INTEGRAL

Inspecting the form of the effective action S_3 , one realizes that the path integral in the canonical partition sum can be easily done in two of the fields, $w(r)$ and $\phi(r)$. Turning

from the ‘Cartesian’ field variables A_{ia} (in mode $j = 0$) to the ‘cylindrical’ variables u, ϕ and w introduced before we use the integration measure

$$\mathcal{D}A_{ia}^{j=0} = \frac{u}{r^3} \mathcal{D}u \mathcal{D}\phi \mathcal{D}w. \quad (29)$$

Because the ansatz (16) is invariant under transformation by \hat{J} , the single monopole partition sum factorizes for small fluctuations around the monopole form:

$$Z_M = \int \mathcal{D}A e^{-S_3[A]} = \int \mathcal{D}A^{j=0} e^{-\beta E[u, \phi, w]} \prod_{j>0} Z_j[u] \quad (30)$$

where

$$Z_j[u] = \int \mathcal{D}(A^j) \exp \left(-S_3 [A^{j=0} + \delta A^j] + S_3 [A^{j=0}] \right) \quad (31)$$

depends only on $u(r)$ due to the gauge freedom (28). Here we have anticipated that we will find a nontrivial stationary point in the mode $u(r)$, around which we can expand the functional integration. All integrations in (31) will be Gaussian except those corresponding to the zero-modes associated with translations and rotations:

$$Z_{j>0} \equiv \prod_{j>0} Z_j[u] = Z_{\text{tr}}[u] Z_{\text{rot}}[u] \prod_{j>0}' Z_j[u], \quad (32)$$

where the prime indicates that the zero-modes have been separated. From studies of small fluctuations around the sphaleron^{18,19} in spontaneously broken SU(2) gauge theory it is known that all other modes are stable, and lead to well-behaved Gaussian integrals in the semiclassical approximation.

The Gaussian integral in the variable w can be easily done. After doing that we get

$$\int \frac{u}{r} \mathcal{D}w \exp \left(-\tilde{\beta} \int_0^\infty dr \left(\frac{w}{r} + \frac{d\phi}{dr} \right)^2 u^2 \right) = \text{const..} \quad (33)$$

compensating for the factor u in (29). We normalize the constant to be unity. Now the compact integral over the field ϕ is trivial and can be normalized such that

$$\int \mathcal{D}\phi = 1, \quad (34)$$

resulting in a functional integral over $u(r)$

$$Z_M = \int \mathcal{D}u \, e^{-\tilde{\beta}E[u]} Z_{\text{tr}}[u] Z_{\text{rot}}[u], \quad (35)$$

if we neglect the influence of non-collective multipole ($j > 0$) fluctuations on the partition function. This issue will be briefly discussed at the end of this Chapter again.

One may wonder at this point why we have chosen these particular normalizations. The physical idea behind it is that for free fields one has to arrive at the canonical partition sum at high temperature. Since the only dynamical degree of freedom is represented by u , the other two fields must not contribute to a correctly normalized path integral.

3.3. SETTING THE SCALE

Unfortunately, $E[u]$ does not have a stable saddle point $u_0(r)$. This lack of a basis for the semiclassical expansion, however, does not preclude the calculation of the partition function for gauge field configurations u with monopole symmetry. It only implies that the evaluation of the functional integral (35) cannot be restricted to integration over Gaussian fluctuations around a classical solution $u_0(r)$. We now propose a method how this functional integral (35), where $u(r)$ satisfies the boundary conditions $u(\infty) = 0$, $u(0) = 1$, may be calculated. It may be practically carried out by adding a stabilizing term to the expression for $E[u]$,

$$E[u] = \int_0^\infty dr \left[\left(\frac{du}{dr} \right)^2 + \frac{(1-u^2)^2}{2r^2} \right], \quad (36)$$

introduced in eq. (24). The idea is to introduce a length scale that favors monopole configurations of a particular core size, and then integrate over the dummy scale in such a way that the partition function remains unchanged. Inspired by the analogous expression for the t'Hooft-Polyakov monopole¹², where a stabilizing mass term $(g\Phi)^2 u^2$ involving the Higgs field Φ appears, we introduce the term $\Delta E[u] = \lambda^2 D[u]$,

$$D[u] = \frac{1}{2} \int_0^\infty dr \, (1 - (1 - u^2)^2), \quad (37)$$

by inserting a unit factor

$$1 = \int_0^\infty d(\lambda^2) \tilde{\beta} D[u] e^{-\tilde{\beta} \lambda^2 D[u]} \quad (38)$$

into the integral (35). Here λ has the dimension of an inverse length. At large r , in view of the boundary condition $u(r) \xrightarrow{r \rightarrow \infty} 0$, it acts like a mass term leading to the asymptotic solution $u(r) \rightarrow A \exp(-\lambda r)$, while it does not interfere with the limit $u(0) = \pm 1$. Our choice of $D[u]$ is unique* if one requests that the integrand be an even function of not higher than fourth order in u which leads to a vanishing energy density at $r \rightarrow \infty$.

After inserting the unit factor (38) into the functional integral (35) we interchange the order of integration over the dummy scale parameter, λ , and with that over the fields. A consistent use of the dimensionless variable $x = \lambda r$ leads now to the following expression:

$$Z_M = \int_0^\infty d\lambda \, 2\tilde{\beta} \int \mathcal{D}u \, D[u] e^{-\tilde{\beta}(E[u] + \Delta E[u])} Z_{\text{tr}} Z_{\text{rot}}, \quad (39)$$

where

$$E[u] = \lambda \int_0^\infty dx \left[\left(\frac{du}{dx} \right)^2 + \frac{(1 - u^2)}{x^2} \right], \quad (40)$$

$$D[u] = \frac{1}{2\lambda} \int_0^\infty dx (2u^2 - u^4), \quad (41)$$

$$\Delta E[u] = \lambda^2 D[u]. \quad (42)$$

* We have explored the consequences of the modified scale breaking factors $D[u] = \frac{1}{N} \int dr (1 - (1 - u^2)^N)$, which are of higher order in $u(r)$. For $N = 3, 4$ these yield somewhat smaller values (by 7 and 14 percent, respectively) for the monopole density within our approximation. This indicates that our result is not very sensitive to the precise form of the scale breaking term. We note that the functional integral over $u(r)$ in (35) could, of course, be performed numerically by Monte-Carlo integration, avoiding the errors introduced in the saddle-point approximation.

The functional integral over u in (39) can now be approximated by a Gaussian integration around the lowest energy stationary solution $u(r) = u_0(x)$ of the exponent, which satisfies the equation

$$\frac{d^2 u_0}{dx^2} + u_0(1 - u_0^2) \left(\frac{1}{x^2} - 1 \right) = 0. \quad (43)$$

Since there is no other scale involved besides the dummy parameter λ , the solution of eq. (43) is solely a function of the dimensionless variable x , and the ground state energy scales as

$$E[u_0] + \Delta E[u_0] = \lambda a. \quad (44)$$

By numerical integration of (43) we have found the value $a = 1.469$. The function $u_0(r)$ is displayed in Figure 1. We also obtain $\Delta E[u_0] = b\lambda$ with $b = 0.695$. The monopole charge contained inside the radius r (eq. 22) is also shown in Fig. 1. The radius $R_{1/2}$ inside which half of the asymptotic charge is contained can be defined as the size of the monopole-soliton. This happens to be the inflection point of the solution $u_0(\lambda r)$

$$R_{1/2} = \lambda^{-1}. \quad (45)$$

Postponing the Gaussian integration over small fluctuations around u_0 we are left with a single-parameter integral:

$$Z_M \approx 2\tilde{\beta} \int_0^\infty d\lambda \, b \, e^{-\tilde{\beta} a \lambda} \, Z_{\text{tr}} Z_{\text{rot}}. \quad (46)$$

3.4. ZERO-MODE CONTRIBUTIONS

Now we turn to the determination of the zero-mode contributions to Z_M . The method we use is that of collective coordinates,²⁰ which leads to an integral over the respective group volumes of translations and rotations, V and $8\pi^2$. What remains is to take into account the normalization of the wave functions describing these modes. Physically they

are related to the total momentum (energy) of a monopole soliton and to its rotational inertia, respectively, as $Z_{\text{tr}} = \lambda^3 V \mathcal{N}_{\text{tr}}^3$, since the classical solution is a function of $x = \lambda r$, and $Z_{\text{rot}} = 8\pi^2 \mathcal{N}_{\text{rot}}^3$, with

$$\mathcal{N}_{\text{tr}}^2 = \frac{1}{6\pi} \int d^3r \, \epsilon(r), \quad (47)$$

$$\mathcal{N}_{\text{rot}}^2 = \frac{1}{6\pi} \int d^3r \, (r^2 \epsilon(r) - \text{correction}), \quad (48)$$

where $\epsilon(r)$ is the energy density of the classical soliton, and the correction—due to an extra gauge rotation—ensures the correct boundary condition for the rotational energy density at radial infinity (see ref. 19). The collective wave function would have the form

$$\varphi = \frac{1}{\sqrt{2E}} e^{iP \cdot x} f, \quad (44)$$

with a form factor f normalized so that including a spin factor 3 for massive solitons and a radial normalization factor 4π we have

$$|\varphi|^2 = \mathcal{N}_{\text{tr}}^{-2} \cdot \frac{1}{2E} \cdot 3 \cdot 4\pi = 1, \quad (50)$$

whence we obtain eq.(47). While the total energy of the soliton scales like $E = 4\pi\lambda a$, the rotational inertia of a homogeneous sphere with radius R like $\frac{3}{5}ER^2 = \frac{3}{5}4\pi a/\lambda$, because the characteristic radius of the soliton is λ^{-1} . This yields the approximate expressions

$$\mathcal{N}_{\text{tr}}^2 \approx \frac{2}{3}\lambda a, \quad (51)$$

and

$$\mathcal{N}_{\text{rot}}^2 \approx \frac{2}{5} \frac{a}{\lambda}, \quad (52)$$

which lead to the zero-mode factors

$$Z_{\text{tr}} = \lambda^3 \overline{Z}_{\text{tr}} = \left(\frac{2}{3}a\right)^{\frac{3}{2}} \lambda^{\frac{3}{2}} \lambda^3 V, \quad (53)$$

$$Z_{\text{rot}} = \left(\frac{2}{5}a\right)^{\frac{3}{2}} \lambda^{-\frac{3}{2}} 8\pi^2. \quad (54)$$

We note that their product scales like λ^3 . The integral over λ in (46) can now be carried out, yielding

$$Z_M = 12 \overline{Z}_{\text{tr}} Z_{\text{rot}} \frac{b}{\tilde{\beta}^3 a^4}. \quad (55)$$

Inserting (53,54) we finally obtain the monopole density in the classical limit as

$$\rho_M = \frac{1}{V} Z_M = \frac{12b}{15^{3/2} \pi a} (g^2 T)^3 = 0.0657 \frac{b}{a} (g^2 T)^3. \quad (56)$$

3.5. RADIAL MONOPOLE FLUCTUATIONS

We finally evaluate the scaled determinantal contribution, $\Delta^{-1/2}$, due to the fluctuations around the soliton-monopole solution $u_0(r)$:

$$\Delta = \prod_n \omega_n^2. \quad (57)$$

Here ω_n^2 are the eigenvalues of the operator

$$\Omega^2 = -\frac{d^2}{dx^2} + (1 - 3u_0^2(x)) \left(1 - \frac{1}{x^2}\right) \quad (58)$$

obtained expanding the effective action $E[u] + \Delta E[u]$ up to second order in $(u - u_0)$. The effective potential involved in this eigenvalue problem,

$$V_{\text{eff}}(x) = (1 - 3u_0^2(x)) \cdot \left(1 - \frac{1}{x^2}\right) \quad (59)$$

is plotted in Figure 2. Since we are calculating the partition sum Z_M , which is restricted to single monopole configurations, we only take into account fluctuations around u_0 that vanish at infinity. Hence only those eigenvalues corresponding to a bound state in this effective potential contribute, i.e.

$$\Delta = \prod_n \omega_n^2 \theta(1 - \omega_n^2) \quad (60)$$

where θ is the step function. Numerically we have found only one such bound state with the eigenvalue $\omega_0^2 = 0.950$.

However, the asymptotic form of V_{eff} , $-\frac{1}{r^2}$, is known to have infinitely many bound states close to threshold. Their eigenvalues, estimated from the Bohr-Sommerfeld formula, are

$$\omega_n^2 = \left(1 - e^{-(2n+1)\pi}\right). \quad (61)$$

The spatial extension of the corresponding eigenfunctions ($n \geq 1$) is so large, that they exceed a radius 70 times larger than the soliton itself. It is therefore questionable whether these fluctuations are physically meaningful and should be taken into account at all for a monopole gas of finite density. Fortunately, this issue is not really critical, since the contribution of such large-size fluctuations is very close to a factor 1

$$\prod_{n=1}^{\infty} \omega_n^2 = \exp\left(\frac{e^{-3\pi}}{1 - e^{-2\pi}}\right) \approx 1.0008. \quad (62)$$

We are left with the contribution of the numerically found ground state in this effective potential which—normalizing the same way as before—reads as

$$\Delta^{-1/2} \approx \frac{1}{\omega_0} \approx 1.03. \quad (63)$$

Collecting all known factors together we obtain the monopole density

$$\rho = 0.0657 \frac{b}{a\omega_0} (g^2 T)^3 \approx 0.0326 (g^2 T)^3, \quad (64)$$

leading to a screening mass of

$$\mu_M \approx 0.255 (g^2 T) \quad (65)$$

for SU(2), which is amazingly close to the value obtained in lattice simulations². Of course, the contributions due to higher multipole modes, which were neglected here (cf. eq. 35), may change this result quantitatively. However, the fact that the determinantal factor for

radial fluctuations is very close to unity lets us be optimistic that the influence of those higher modes on the numerical constant in eq. (65) will be minor.

4. Conclusions

Finally we would like to present some numerical estimates characteristic for QCD under conditions that may be reached in nuclear collisions at RHIC energies. We assume a temperature of $T = 300$ MeV and a coupling constant of $g = 2$, corresponding to $\alpha_s = g^2/4\pi \approx 0.32$. We note that while this value of α_s would justify a perturbative QCD approach, the monopole charge $g^{-1} = 0.5$ may justify the neglect of monopole-monopole interactions. The magnetic screening mass for SU(3) we obtain using the above values is $\mu_M \approx 375$ MeV yielding a screening length of $\mu_M = 0.53$ fm for static magnetic fields.

The size and the total energy of an average monopole-soliton can also be estimated using the scale representation of eq. (46):

$$\rho_M = \int_0^\infty d\lambda \, n(\lambda) \, e^{-\lambda a \tilde{\beta}} \quad (66)$$

with $n(\lambda) = n_0 \lambda^3$. The average soliton size is obtained as

$$R_M = \langle \lambda^{-1} \rangle = \frac{1}{3} a \tilde{\beta} \approx \frac{6.10}{g^2 T}, \quad (67)$$

which is about 1 fm at this temperature. Unfortunately this exceeds the average distance $d_0 \approx 4/g^2 T$ between monopoles at the equilibrium density (64) indicating a possible breakdown of the dilute gas approximation employed here. On the other hand, the average interaction energy between monopoles

$$E_{\text{int}} = (4\pi g^2 d_0)^{-1} \approx 0.02 \, T \quad (68)$$

is quite small compared with the average monopole mass

$$\langle E \rangle = E[u_0]/g^2 = 4 \, T. \quad (69)$$

This indicates that higher-order terms in the effective action $S_3[A]$ for superpositions of monopoles are small, and gives reason to believe that the dilute gas approximation may be quite trustworthy. It allows for an intuitive understanding of the mechanism for static magnetic screening in thermal gauge theories. We hope to return to the problem of evaluating the determinant of eigenvalues of higher modes, as well as to the question of non-static loop corrections in the future.

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Appendix A

In this appendix we derive the color charge density (eq.6) from the dilute gas partition function (4) in the weak external field limit. From eq.(4) we obtain the following expression for the logarithm of the partition function

$$\ln Z = - \sum_{\omega} \sum_{q_3} \ln (1 - e^{-\beta\omega} e^{\Theta q_3}), \quad (A.1)$$

which after factorizing the color charge independent one-gluon partition functions can be casted into the form

$$\ln Z = - \sum_{\omega} \sum_{q_3} [\ln (1 - e^{-\beta\omega}) + \ln (1 + n(\omega) (1 - e^{\Theta q_3}))], \quad (A.2)$$

where

$$n(\omega) = \frac{1}{e^{\beta\omega} - 1} \quad (A.3)$$

is the Bose-Einstein distribution function.

Restoring the generality of the discussion we replace now Θq_3 again by $\beta\phi^a \hat{Q}^a$ and write the summation over q_3 as the adjoint color trace tr . We arrive at

$$\ln Z = - \sum_{\omega} \left[\ln (1 - e^{-\beta\omega}) tr 1 + tr \ln (1 + n(\omega) (1 - e^{\beta\phi^a \hat{Q}^a})) \right]. \quad (A.4)$$

Separating now the external field independent term,

$$\ln Z_0 = -(N^2 - 1) \sum_{\omega} \ln (1 - e^{-\beta\omega}), \quad (A.5)$$

we are left with

$$\ln Z = \ln Z_0 - \sum_{\omega} tr \ln (1 + n(\omega) (1 - e^{\beta\phi^a \hat{Q}^a})), \quad (A.6)$$

which can be approximated in the dilute limit $n(\omega) \ll 1$ by

$$\ln Z \approx \ln Z_0 - \sum_{\omega} n(\omega) tr (1 - e^{\beta\phi^a \hat{Q}^a}). \quad (A.7)$$

Finally we use the fact that the external field ϕ^a is infinitesimal in case of seeking for a linear response of the gluon medium. We obtain

$$\ln Z \approx \ln Z_0 + \sum_{\omega} n(\omega) \operatorname{tr} \left(\beta \phi^a \hat{Q}^a + \frac{\beta^2}{2} \phi^a \phi^b \hat{Q}^a \hat{Q}^b + \dots \right), \quad (\text{A.8})$$

which, normalizing the color magnetic charge generators according to the adjoint representation of the $\text{SU}(N)$ algebra,

$$\operatorname{tr} \left(\hat{Q}^a \hat{Q}^b \right) = -\frac{N}{g^2} \delta^{ab}, \quad (\text{A.9})$$

leads to

$$\ln Z \approx \ln Z_0 - \frac{\beta^2 N}{2g^2} \phi^a \phi^a \sum_{\omega} n(\omega). \quad (\text{A.10})$$

The definition (5) of the color charge density leads finally to

$$\rho_E^a = \frac{1}{\beta V} \frac{\partial}{\partial \phi^a} \ln Z \approx -\frac{N}{V g^2 T} \phi^a \sum_{\omega} n(\omega), \quad (\text{A.11})$$

which is equivalent to eqs. (6) and (7).

To calculate the screening of static electric fields, one replaces the scalar magnetic potential by a Coulomb potential

$$E_i^a = -\partial_i \phi_E^a, \quad (\text{A.12})$$

and the magnetic charge operator \hat{Q}^a by the electric charge operator \hat{Q}_E^a . The sole change in the derivation following equation (A.1) then is that (A.9) is now replaced by

$$\operatorname{tr} \left(\hat{Q}_E^a \hat{Q}_E^b \right) = -N g^2 \delta^{ab}. \quad (\text{A.13})$$

We finally obtain for the induced color-electric charge density:

$$\rho_E^a \approx -\frac{N g^2}{V T} \phi_E^a \sum_{\omega} n(\omega). \quad (\text{A.14})$$

Using the free gluon dispersion relation we have

$$\sum_{\omega} n(\omega) = V \int \frac{d^3k}{(2\pi)^3} \left(e^{\beta|k|} - 1 \right)^{-1} = \frac{1}{3} V T^3, \quad (A.15)$$

hence

$$\rho_E^a = -\frac{N}{3} g^2 T^2 \phi^a \equiv -\mu_E^2 \phi^a. \quad (A.16)$$

This is the standard perturbative result for the static electric screening mass in thermal Yang-Mills theories.

Appendix B

In this appendix we derive the expressions (21-24) for the magnetic field and energy from the ansatz (16) for the vector potential. It is useful here to introduce some short-hand notations. We write

$$A_{ia} = A^+ P_{ia}^+ + A^- P_{ia}^- + A^0 P_{ia}^0. \quad (B.1)$$

For calculating derivatives of this expression we start with the projectors. First we note that

$$\partial_j n_i = \frac{1}{r} (\delta_{ij} - u_i u_j) \quad (B.2)$$

and

$$\delta_j (n_i n_k) = \frac{1}{r} (\delta_{ij} n_k + \delta_{jk} n_i - 2 n_i n_k n_j). \quad (B.3)$$

It is easy to obtain then

$$\epsilon_{ijk} \partial_j (\epsilon_{kal} n_l) = \frac{1}{r} (\delta_{ia} + n_i n_a) \quad (B.4)$$

and

$$\epsilon_{ijk} \partial_j (n_k n_a) = \frac{1}{r} \epsilon_{iak} n_k. \quad (B.5)$$

Using this we obtain the following expressions for the curls of the projectors

$$\epsilon_{ijk} \partial_j P_{ka}^+ = -\frac{1}{2r} \epsilon_{iak} n_k + \frac{i}{2r} (\delta_{ia} + u_i n_a) = \frac{i}{r} (P_{ia}^+ + P_{ia}^0), \quad (B.6)$$

$$\epsilon_{ijk} \partial_j P_{ka}^- = -\frac{1}{2r} \epsilon_{iak} n_k - \frac{i}{2r} (\delta_{ia} + n_i n_a) = \frac{i}{r} (P_{ia}^- + P_{ia}^0) \quad (B.7)$$

and

$$\epsilon_{ijk} \partial_j P_{ka}^0 = \frac{1}{r} \epsilon_{iak} n_k = -\frac{i}{r} (P_{ia}^+ - P_{ia}^-). \quad (B.8)$$

It is now easy to decompose the curl of the vector potential remembering that ∂_j acts on pure radial functions as $n_j \frac{d}{dr}$. Using

$$\epsilon_{ijk} n_j P_{ka}^+ = -\frac{1}{2} \epsilon_{iaj} n_j + \frac{i}{2} (\delta_{ia} - n_i n_a) = i P_{ia}^+, \quad (B.9)$$

$$\epsilon_{ijk}n_j P_{ka}^- = -iP_{ia}^-, \quad (B.10)$$

and

$$\epsilon_{ijk}n_j P_{ka}^0 = 0 \quad (B.11)$$

we get

$$\epsilon_{ijk}\partial_j A_{ka} = iP_{ia}^+ \left(\frac{dA^+}{dr} + \frac{1}{r}A^+ - \frac{1}{r}A^0 \right) + c.c + iP_{ia}^0 \left(\frac{1}{r}A^+ - \frac{1}{r}A^- \right). \quad (B.12)$$

We note that $(A^+)^* = A^-$, $(P_{ia}^+)^* = P_{ia}^-$.

After evaluating the abelian part of the magnetic field (eq. 21) we turn to the calculation of the nonabelian part

$$N_{ia} = \frac{1}{2}\epsilon_{abc}\epsilon_{ijk}A_{jb}A_{kc}. \quad (B.13)$$

Noting that

$$\epsilon_{abc}\epsilon_{ijk} = \det \begin{vmatrix} \delta_{ai} & \delta_{aj} & \delta_{ak} \\ \delta_{bi} & \delta_{bj} & \delta_{bk} \\ \delta_{ci} & \delta_{cj} & \delta_{ck} \end{vmatrix} \quad (B.14)$$

we obtain

$$N_{ia} = \frac{1}{2}\delta_{ai} (A_{jj}A_{kk} - A_{jj}^2) - A_{kk}A_{ai} + A_{ai}^2. \quad (B.15)$$

Now it is straightforward to evaluate A_{ai}^2 and its traces, using the projectors. We get

$$A_{jj} = A^+ + A^- + A^0, \quad (B.16)$$

$$(A^2)_{ai} = A^+A^+P_{ai}^+ + A^-A^-P_{ai}^- + A^0A^0P_{ai}^0 \quad (B.17)$$

and

$$(A^2)_{jj} = (A^+)^2 + (A^-)^2 + (A^0)^2.$$

Inserting these expressions into (B.15) we get

$$N_{ia} = \delta_{ai}(A^+A^- + A^-A^0 + A^0A^+) - A^+(A^- + A^0)P_{ai}^+ - A^-(A^+ + A^0)P_{ai}^0 - A^0(A^+ + A^-)P_{ai}^0. \quad (B.18)$$

Noting that $P_{ai}^+ = P_{ia}^-$, $P_{ai}^- = P_{ia}^+$ and $\delta_{ia} = P_{ia}^+ + P_{ia}^- + P_{ia}^0$, we finally get the simple result

$$N_{ia} = A^+ A^0 P_{ia}^+ + A^- A^0 P_{ia}^- + A^+ A^- P_{ia}^0. \quad (B.19)$$

The definition of the magnetic field can now be used to obtain the decomposition

$$B_{ia} = B^+ P_{ia}^+ + B^- P_{ia}^- + B^0 P_{ia}^0. \quad (B.20)$$

We get

$$B^+ = i \frac{d}{dr} A^+ + \frac{i}{r} A^+ - \frac{i}{r} A^0 - A^+ A^0, \quad (B.21)$$

$$B^- = -i \frac{d}{dr} A^- - \frac{i}{r} A^- + \frac{i}{r} A^0 - A^- A^0 \quad (B.22)$$

and

$$B^0 = \frac{i}{r} A^+ - \frac{i}{r} A^- - A^+ A^-. \quad (B.23)$$

Now we include the trivial factor $\frac{1}{r}$ in the definition of the vector potential using the notation $A^+ = \frac{1}{r} a^+$, etc. We get

$$B^+ = \frac{1}{r} \left(i \frac{d}{dr} a^+ - \frac{1}{r} a^0 (i + a^+) \right), \quad (B.24)$$

$$B^- = \frac{1}{r} \left(-i \frac{d}{dr} a^- + \frac{1}{r} a^0 (i - a^-) \right) \quad (B.25)$$

and

$$B^0 = \frac{1}{r^2} (i a^+ - i a^- - a^+ a^-). \quad (B.26)$$

One realizes at this point that the form of the ansatz (16) is simple in terms of the magnetic field giving

$$a^+ + i = u e^{i\phi}, \quad (B.27)$$

$$a^- - i = u e^{-i\phi}, \quad (B.28)$$

$$a^0 = w. \quad (B.29)$$

Using this we arrive at

$$a^+ - a^- = 2iu \sin \phi - 2i, \quad (B.30)$$

$$a^+ a^- = u^2 + iue^{i\phi} - iue^{-i\phi} + 1 = 1 - 2u \sin \phi + u^2 \quad (B.31)$$

whence finally we obtain

$$B^+ = \frac{1}{r} \left(i \frac{du}{dr} e^{i\phi} - u \frac{d\phi}{dr} e^{i\phi} - \frac{1}{r} w u e^{i\phi} \right), \quad (B.32)$$

$$B^- = \frac{1}{r} \left(-i \frac{du}{dr} e^{-i\phi} - u \frac{d\phi}{dr} e^{-i\phi} - \frac{1}{r} w u e^{-i\phi} \right) \quad (B.33)$$

and

$$B^0 = \frac{1}{r^2} (1 - u^2). \quad (B.34)$$

To obtain the magnetic flux and the magnetic charge is easy observing that P_{ia}^+ and P_{ia}^- are orthogonal to n_i . So we arrive at eq. (22)

$$n_i B_{ia} = n_a (1 - u^2). \quad (B.35)$$

In order to calculate the energy density we note that since $B^- = (B^+)^*$, $P_{ia}^- = (P_{ia}^+)^*$,

$$\mathcal{E} = \frac{1}{2} B_{ia} B_{ia} = B^+ \cdot B^- + \frac{1}{2} B^0 \cdot B^0 \quad (B.36)$$

is whence

$$\mathcal{E} = |B^+|^2 + \frac{1}{2} B^0 B^0 = \frac{1}{r^2} \left| i \frac{du}{dr} - u \left(\frac{d\phi}{dr} + \frac{w}{r} \right) \right|^2 + \frac{1}{2r^4} (1 - u^2)^2. \quad (B.37)$$

Finally it gives rise to the following reduced dimensional action

$$S_3 = \frac{1}{g^2 T} E = \frac{4\pi}{g^2 T} \int_0^\infty dr \left[\left(\frac{du}{dr} \right)^2 + u^2 \left(\frac{d\phi}{dr} + \frac{w}{r} \right)^2 + \frac{1}{2r^2} (1 - u^2)^2 \right] \quad (B.38)$$

as presented in eq. (24).

It remains to calculate the action of an infinitesimal gauge transformation on the monopole ansatz. From the general expression

$$\delta A_{ia} = \mathcal{D}_{iab} \Lambda_b \quad (B.39)$$

we conclude that $\Lambda_b = n_b \cdot \Lambda(r)$ is the only form not leaving the ansatz's configuration space, because only the derivative of n_i is expressable through a combination of P_{ij}^\pm and P_{ij}^0 . Noting that

$$D_{iab} = \delta_{ab}\partial_i - \epsilon_{acb}A_{ic} \quad (B.40)$$

and

$$\epsilon_{acb}n_b = -iP_{ac}^+ + iP_{ac}^- \quad (B.41)$$

we obtain

$$D_{iab}n_b\Lambda = n_in_a\frac{d\Lambda}{dr} + \frac{1}{r}(\delta_{ia} - n_in_a)\Lambda - i(P_{ac}^+ - P_{ac}^-)A_{ci} \quad (B.42)$$

which, upon using the generic form (A.1) for the vectorpotential, yields

$$\delta A^+ = \frac{1}{r}\Lambda - iA^+ \cdot \Lambda, \quad (B.43)$$

$$\delta A^- = \frac{1}{r}\Lambda + iA^- \cdot \Lambda \quad (B.44)$$

and

$$\delta A^0 = \frac{d}{dr}\Lambda.$$

Replacing now the expressions for A^\pm and A^0 we finally arrive at

$$\delta u = 0, \quad (B.45)$$

$$\delta\phi = -\Lambda \quad (B.46)$$

and

$$\delta w = r\frac{d}{dr}\Lambda, \quad (B.47)$$

as claimed in eqs. (28).

Appendix C

In this appendix we briefly describe the methods we employed to obtain the numerical results of this paper.

The classical monopole-soliton as a solution of eq. (43) can not be obtained by direct numerical integration, because the fixed boundary condition $u(\infty) = 0, u(0) = 1$. Instead we applied the energy-functional relaxation updating the field $u_n = u_0(x)$ known on a lattice of $x_n = \lambda \cdot n \cdot \Delta r = n \cdot \Delta x$ points due to the conjugate gradient method

$$u'_n = u_n - \epsilon \frac{\delta E[u]}{\delta u_n} \quad (C.1)$$

using $\epsilon = 0.01$, $\Delta x = 0.01 - 0.05$ and 200 - 1000 grid points. We obtained the same solution by a second order conjugate gradient method.

We also found the eigenvalues of the effective potential (59) for monopole-like fluctuations numerically. The first method employed a “shooting” algorithm, integrating from $x = 0$ and from $x = \infty$ up to a matching point, and modifying the initial derivatives in order to fit the logarithmic derivative at the matching. The second method directly located the zeros of the determinant of the discretized matrix

$$\Omega_{mn}^2 - \omega^2 \Delta x^2 \delta_{mn} = -\delta_{n,m+1} - \delta_{n,m-1} + \delta_{mn} (2 + \Delta x^2 (V_{\text{eff}}[U_n, n] - \omega^2)) \quad (C.2)$$

in the interval $0 \leq \omega \leq 1$. The evaluation of the determinant made use of the simple recursion formula for tri-diagonal $N \times N$ matrices

$$D_N = q_N \cdot D_{N-1} - D_{N-2}, \quad (C.3)$$

where q_N is the N -dependent diagonal element and $D_0 = 1$, $D_1 = q_1$ start the recurrence.

Figure Captions

Fig. 1 The classical magnetic monopole soliton $u_0(r)$ which minimizes the energy functional $E[u] + \Delta E[u]$ is shown as solid line. The resulting monopole charge contained inside a sphere of radius r is represented by the dashed line.

Fig. 2 The effective potential for radial magnetic monopole fluctuations $\delta u(r)$ is plotted as a function of the scaled radius $x = \lambda r$. The ground state energy $\omega_0^2 = 0.95$ in this potential is indicated by a horizontal straight line.